FULLY MICROSCOPIC CALCULATIONS FOR CLOSED-SHELL NUCLEI WITH REALISTIC NUCLEON-NUCLEON POTENTIALS

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The ground-state energy of the doubly magic nuclei ^{4}He and ^{16}O has been calculated within the framework of the Goldstone expansion starting from modern nucleon-nucleon potentials. A low-momentum potential $V_{\mathrm{low-k}}$ has been derived from the bare potential by integrating out its high-momentum components beyond a cutoff Λ. We have employed a simple criterion to relate this cutoff momentum to a boundary condition for the two-nucleon model space spanned by a harmonic-oscillator basis. Convergence of the results has been obtained with a limited number of oscillator quanta.

1. Introduction

As is well known, the strong repulsive components in the high-momentum regime of a realistic nucleon-nucleon (NN) potential V_{NN} need to be renormalized in order to perform perturbative nuclear structure calculations. In Refs. [1,2] a new method to renormalize the NN interaction has been proposed, which consists in deriving an effective low-momentum potential $V_{\mathrm{low-k}}$ that satisfies a decoupling condition between the low- and high-momentum spaces. This $V_{\mathrm{low-k}}$ preserves exactly the on-shell properties of the original V_{NN} up to a cutoff momentum Λ, and is a smooth potential which can be used directly in nuclear structure calculations.

In the past few years, we have employed this approach to calculate the ground-state (g.s.) properties of doubly closed-shell nuclei within the framework of the Goldstone expansion [3,4], using a fixed value of the cutoff momentum.

Recently, we have investigated how the cutoff momentum Λ is related to the dimension of the configuration space in the coordinate representation [5], where our calculations are performed. We have shown how the choice

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of a cutoff momentum corresponds to fix a boundary for the two-nucleon model space.

In the present work, we calculate the g.s. energy of ^{4}He and ^{16}O in the framework of the Goldstone expansion with different NN potentials. To verify the validity of our approach, we compare the ^{4}He results with those obtained using the Faddeev-Yakubovsky (FY) method.

The paper is organized as follows. In Sec. 2 we give a brief description of our calculations. Sec. 3 is devoted to the presentation and discussion of our results for ^{4}He and ^{16}O. A summary of our study is given in Sec. 4.

2. Outline of calculations

As mentioned in the Introduction, the short-range repulsion of the NN potential is renormalized integrating out its high-momentum components through the so-called $V_{\mathrm{low}-\mathrm{k}}$ approach (see Refs. [1,2]). The $V_{\mathrm{low}-\mathrm{k}}$ preserves the physics of the two-nucleon system up to the cutoff momentum Λ. While this low-momentum potential is defined in the momentum space, we perform our calculations for finite nuclei in the coordinate space employing a truncated HO basis. This makes it desirable to map the cutoff momentum Λ, which decouples the momentum space into a low- and high-momentum regime, onto a boundary for the HO space [5].

If we consider the two-nucleon relative motion in a HO well in the momentum representation, then, for a given maximum relative momentum Λ, the corresponding maximum value of the energy is

$$E_{\max} = \frac{\hbar^2 \Lambda^2}{M} \quad , \tag{1}$$

where M is the nucleon mass.

We rewrite this relation in the relative coordinate system in terms of the maximum number $N_{\max}$ of HO quanta:

$$\left(N_{\max} + \frac{3}{2} \right) \hbar\omega = \frac{\hbar^2 \Lambda^2}{M} \quad , \tag{2}$$

for a given HO parameter $\hbar\omega$. The above equation provides a simple criterion to map out the two-nucleon HO model space. Let us write the two-nucleon states as the product of HO wave functions

$$|a\,b\rangle = |n_a l_a j_a,\ n_b l_b j_b\rangle \quad . \tag{3}$$

We define our HO model space as spanned by those two-nucleon states that satisfy the constraint

$$2n_a + l_a + 2n_b + l_b \leq N_{\max} \quad . \tag{4}$$

In this paper, making use of the above approach, we have calculated the g.s. energies of ^{4}He and ^{16}O within the framework of the Goldstone expansion [6]. We start from the intrinsic hamiltonian

$$H = \left(1 - \frac{1}{A}\right) \sum_{i=1}^{A} \frac{p_i^2}{2M} + \sum_{i<j} \left(V_{ij} - \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{MA}\right) \quad , \tag{5}$$

where V_{ij} stands for the renormalized V_{NN} potential plus the Coulomb force, and construct the Hartree-Fock (HF) basis expanding the HF single particle (SP) states in terms of HO wave functions. The following step is to sum up the Goldstone expansion including contributions up to fourth-order in the two-body interaction. Using Padé approximants [7,8] one may obtain a value to which the perturbation series should converge. In this work, we report results obtained using the Padé approximant [2|2], whose explicit expression is

$$[2|2] = \frac{E_0(1 + \gamma_1 + \gamma_2) + E_1(1 + \gamma_2) + E_2}{1 + \gamma_1 + \gamma_2} \quad , \tag{6}$$

where

$$\gamma_1 = \frac{E_2 E_4 - E_3^2}{E_1 E_3 - E_2^2} \quad , \qquad \gamma_2 = -\frac{E_3 + E_1 \gamma_1}{E_2} \quad ,$$

E_i being the ith order energy contribution in the Goldstone expansion.

Our calculations are made in a truncated model space, whose size is related to the values of the cutoff momentum Λ and the $\hbar\omega$ parameter. The calculations are performed increasing the $N_{\max}$ value (and consequently Λ) and varying $\hbar\omega$ until the dependence on $N_{\max}$ (Λ) is minimized.

3. Results

We have calculated the binding energy of ^{4}He using different V_{NN}'s, and compared our results with those obtained by means of the FY method. This comparison is made in order to test the reliability of our approach. In Figs. 1, 2, and 3 the calculated ^{4}He g.s. energies obtained from the CD-Bonn [9],

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N^3LO [10], and Bonn A [11] NN potentials are reported, for different values of $\hbar\omega$, as a function of the maximum number N_{max} of HO quanta. The FY result [12,13] is also shown.
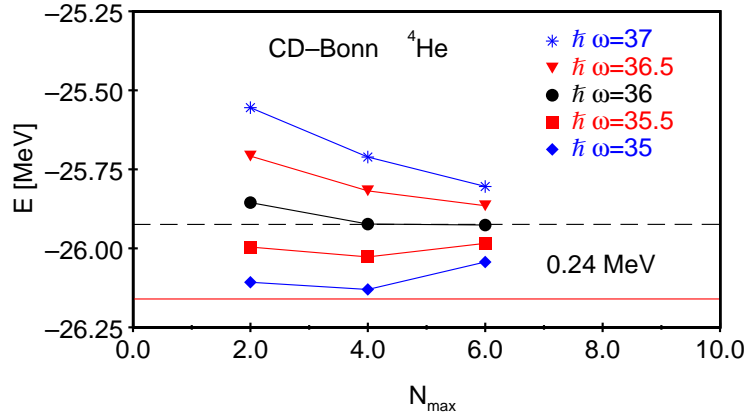

Fig. 1.   Ground state energy of ^{4}He calculated with the CD-Bonn potential as function of N_{max}, for different values of $\hbar\omega$. The straight line represents the Faddeev-Yakubovsky result, while the dashed one our converged result. The difference in energy between the latter and the Faddeev-Yakubovsky result is also reported.
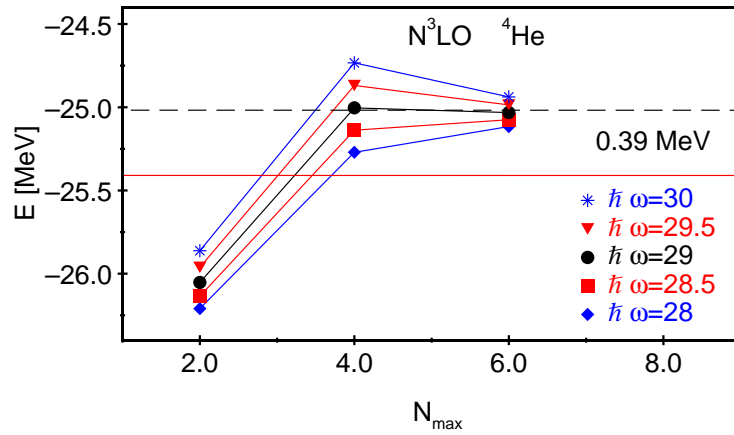

Fig. 2.   Same as Fig. 1, but for the N^3LO potential.

For the sake of clarity, in Table 1 we report the numerical values obtained with the CD-Bonn potential. From the inspection of Table 1 it can be seen
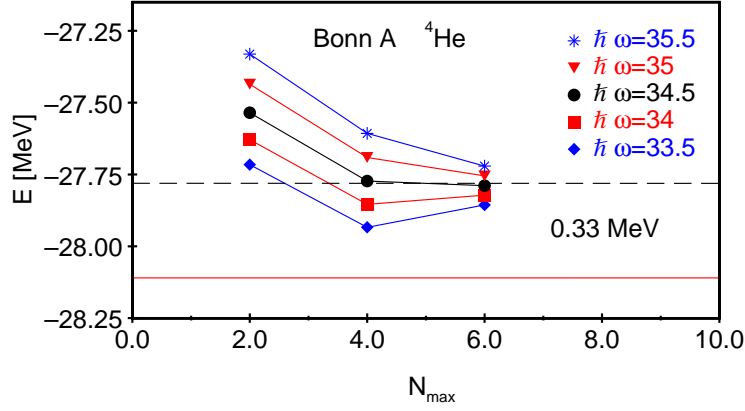
Fig. 3.   Same as Fig. 1, but for the Bonn A potential.

that the g.s. energy does not change increasing $N_{\max}$ from 4 to 6 for $\hbar\omega = 36$ MeV. On these grounds, we choose as our final result that corresponding to the above $\hbar\omega$ value, i.e. -25.92 MeV. Moreover, we find it worthwhile to introduce a theoretical error due to the uncertainty when choosing $\hbar\omega_{\mathrm{conv}}$, which corresponds to the one with the faster convergence with $N_{\max}$. We estimate this error as the largest difference in energy between the final result and those corresponding to the two $\hbar\omega$ values adjacent to $\hbar\omega_{\mathrm{conv}}$. For the CD-Bonn potential, we see that this difference is 0.05 MeV for the largest $N_{\max}$.

Table 1.   Ground state energy of ^{4}He (in MeV) calculated with the CD-Bonn potential for different values of $\hbar\omega$ and $N_{\max}$

$\hbar\omega$ (in MeV)	$N_{\max} = 2$	$N_{\max} = 4$	$N_{\max} = 6$
35.0	-26.11	-26.13	-26.04
35.5	-25.10	-26.03	-25.97
36.0	-25.86	-25.92	-25.92
36.5	-25.71	-25.83	-25.87
37.0	-25.56	-25.71	-25.80

Similarly, the results for the ^{4}He g.s. energy with the N^{3}LO and the Bonn A potentials are (-25.02 ± 0.05) and (-27.78 ± 0.03) MeV, respectively. These values are in good agreement with the FY results, the largest discrepancy being 0.39 MeV for N^{3}LO potential.
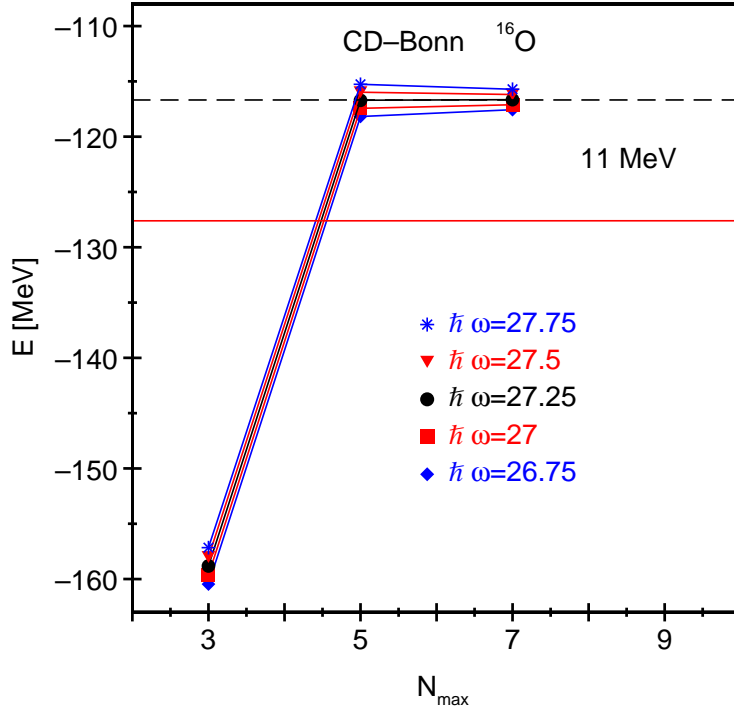
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Fig. 4.   Ground state energy of ^{16}O calculated with the CD-Bonn potential as function of $N_{\max}$, for different values of $\hbar\omega$. The straight line represents the experimental value [14], while the dashed one our converged result. The difference in energy between the latter and the experimental value is also reported.

We have also calculated the g.s. energy of ^{16}O starting from both the CD-Bonn and the Bonn A potential, as reported in Figs. 4 and 5, respectively. With the CD-Bonn potential, the converged value, obtained for $\hbar\omega = 27.25$ MeV, is equal to (-117 ± 1) MeV, the discrepancy with the experimental value [14] being 11 MeV. This value is slightly different $(\approx 1$ MeV$)$ from the one reported in our previous paper [5], because in the present work we have decreased by a factor 2 the spacings between the $\hbar\omega$ values. It is worth to point out that this result is consistent with those obtained by Fujii *et al.* using the unitary model-operator approach [15], and by Vary *et al.* in the no-core shell model framework [16].

A better agreement with experiment is obtained using the weaker tensor force NN potential Bonn A, our ^{16}O g.s. energy being (-130.0 ± 0.5) MeV.
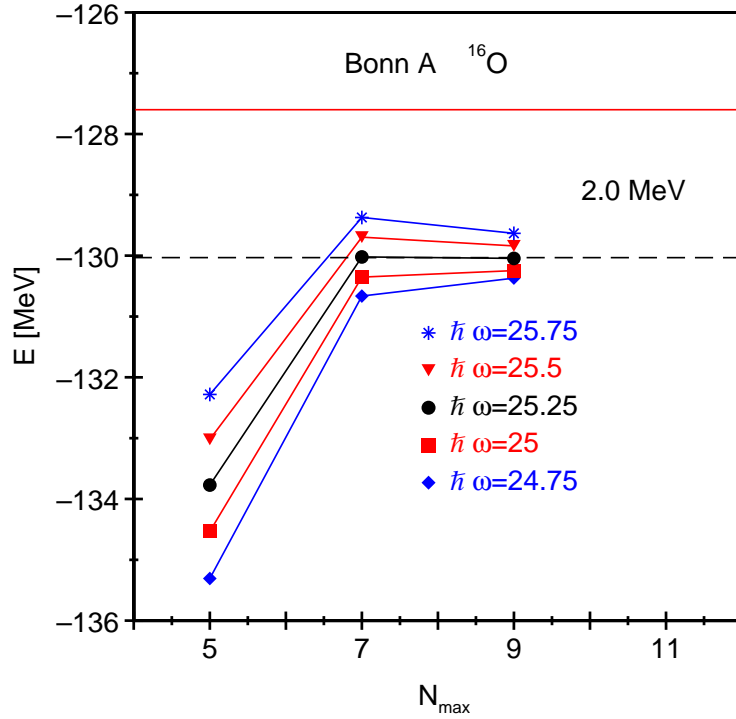
Fig. 5.   Same as Fig. 4, but for the Bonn A potential.

4. Summary

In this work, we have calculated the g.s. energy of the doubly closed-shell nuclei ^{4}He and ^{16}O in the framework of the Goldstone expansion, starting from different realistic NN potentials. In order to renormalize their short-range repulsion, the high-momentum components of these potentials have been integrated out through the so-called V_{low-k} approach. We have employed a criterion to map out the model space of the two-nucleon states in the HO basis according to the value of the cutoff momentum Λ [5].

To show the validity of this procedure, we have calculated the g.s. energy of ^{4}He, with the CD-Bonn, N^{3}LO, and Bonn A potentials, comparing the results with the FY ones. We have found that the energy differences do not exceed 0.39 MeV. The limited size of the discrepancies evidences that our approach provides a reliable way to renormalize the NN potentials preserving not only the two-body but also the many-body physics.

On the above grounds, we have performed similar calculations for ^{16}O

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with the CD-Bonn and Bonn A NN potentials, and obtained converged results using model spaces not exceeding $N_{\max} = 9$. The rapid convergence of the results with the size of the HO model space makes it very interesting to study heavier systems employing our approach [17].

5. Acknowledgments

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